Phil's Orderly Physics Curriculum Important Concepts List (POPCICL) 1C

[Warning : This list is NOT intended to be comprehensive, but rather to highlight a few key concepts]

Oscillatory Motion

A repeated back-and-forth motion is called an oscillation

Oscillations can arise when you have a system that provides a restoring force that tend to restore the system back towards its equilibrium position.

Simple harmonic motion is the name given to oscillations where the motion can be described mathematically by sinusoids (sines or cosines). A system that undergoes simple harmonic motion (SHM) is called a simple harmonic oscillator (SHO).

The time that it takes to complete one full oscillation is called the period, T, of the motion. The number of full oscillations per second is call the frequency, f.

The position for the sinusoidal motion of a SHO can be written as $x = A\cos(\omega t + \phi)$ where A is the amplitude of the oscillation, ω is the angular frequency of the oscillation and ϕ is the phase constant.

The amplitude is the positive maximal displacement of the object from its equilibrium position.

The angular frequency is measured in radians per second. It describes how the sinusoidal back-and-forth motion of a SHO is related to the projection (or "shadow") of uniform circular motion onto a 1-D axis.

The phase factor determines shifts the sinusoidal function back-or-forward in time to allow for systems that are in a particular portion of their oscillatory cycle (peak, trough, or anywhere in between) at time t=0. [Note that $\cos(\omega t + \pi/2)$ is identical to $\sin(\omega t)$]

The frequency and period of an oscillating mass on a spring depend on the spring constant and the mass, but do not depend on the amplitude of the oscillation.

The velocity for the sinusoidal motion of a SHO is the time derivative of the position, and is also sinusoidal. $v = -\omega A \sin(\omega t + \phi)$

The acceleration for the sinusoidal motion of a SHO is the time derivative of the velocity, and is also sinusoidal. $a = -\omega^2 A \cos(\omega t + \phi)$

An object undergoing SHM has a maximum speed, and hence maximum kinetic energy at the equilibrium position, $|\Delta x| = 0$. The speed and kinetic energy drop to zero at the turning points, where $|\Delta x| = A$

An object undergoing SHM has a maximum net force, and hence the magnitude of the acceleration is also maximal at the turning points, where $|\Delta x| = A$. The net force on the object and its acceleration drop to zero at the equilibrium position, $|\Delta x| = 0$

As the SHO oscillates, the total energy remains constant, but the distribution of that energy between potential energy and kinetic energy oscillates over time.

At the turning points, where $|\Delta x| = A$, the kinetic energy drops to zero, and so all of the total energy is stored as potential energy; thus the total energy of the SHO is given by : $E = \frac{1}{2}kA^2$.

A simple pendulum can be approximated as a SHO for small angle oscillations.

The frequency and period of an simple pendulum depend on the length of the pendulum and the magnitude of the gravitational force (which is nearly constant anywhere on the surface of the Earth); but they are independent of both the mass at the end of the pendulum, and the amplitude of the oscillation.

Mechanical Waves

A wave is a disturbance that propagates (moves in a particular direction) away from its source.

A mechanical wave is a wave that requires a physical medium through which to propagate. A non-mechanical wave (such as an electromagnetic wave) does not require a medium (it can propagate through empty space)

A transverse wave is a wave in which the direction of the local disturbance is perpendicular to the direction of the wave propagation. A wave on a string is an example of a transverse wave.

A longitudinal wave is a wave in which the direction of the local disturbance is parallel to the direction of the wave propagation. A sound wave is an example of a longitudinal wave.

A one-dimensional sinusoidal wave traveling in the positive x direction can be described by the wave function: $y = A \sin (kx - \omega t)$, where A is the amplitude, k is the wave number, and ω is the angular frequency.

The wave number, k, is related to the wavelength, λ, by the relationship : $k = 2\pi / \lambda$.

The angular frequency, ω, is related to the frequency, f, and period, T, by the relationship : $\omega = 2\pi f = 2\pi / T$

The wavelength is the separation (across space) of two identical points on a periodic (repeating) propagating wave.

The period is the separation (across time) of two identical points of a periodic propagating wave

The frequency is a measure of the number of full cycles of the wave are completed per second.

The angular frequency is a measure of the number of radians of phase are completed per second.

There are 2π radians of phase in a full cycle (the circumference of a unit circle is 2π)

The speed of a wave is related to the velocity of that wave through a particular medium by the “wave speed relationship” : $v = \lambda f$.

The speed of a wave (including the speed of sound) is purely a property of the material. It does not depend on the amplitude or frequency of the wave. (In the same air, all sounds will travel at the same speed regardless of how loud they are or how high-or-low pitched they are).

In general, the speed of a wave in a material is proportional to the square-root of the ratio of the media's elastic properties to its inertia properties.

For a wave on a string, the wave speed is equal to the square-root of the ratio of the Tension in the string to the Mass per unit length of the string.

For a sound wave, the wave speed is equal to the square-root of the ratio of the Bulk modulus of the medium to the volume density of the medium.

The bulk modulus is a measure of “how difficult” it is to compress the medium. A medium that is very difficult to compress has a large bulk modulus, and thus a higher wave speed.

The density of a material (including air) depends on its temperature, and so the speed of sound in the material will also depend on the temperature.

If the source of a sound and the observer of that sound are both stationary, then the observer will hear the exact same frequency as the original source.

If the source or the observer are moving relative to each other, the observer will hear a different frequency than the original source. The observed frequency will be shifted higher if the relative motion is bringing them closer together, and the observed frequency will shifted lower if the relative motion is taking them further apart.

When a wave crosses a boundary from a lower density material to a higher density material (such as a rigid wall or a heavier string) then at least some of the wave will be reflected back with its amplitude inverted.

When a wave crosses a boundary from a higher density material to a lower density material (such as a free end or a lighter string) then at least some of the wave will be reflected back with its amplitude upright.

When two traveling waves encounter each other, they can momentarily exhibit constructive or destructive interference, but then continue to propagate past their point of overlap unaltered.

Standing Waves

Two overlapping waves that are counter-propagating (moving in opposite directions) but have the same frequency can interfere to form a standing wave.

A sinusoidal standing wave has the form : $y = [2A \sin(kx)]\cdot\cos(\omega t)$, where A, k and ω are the amplitude,wave number and angular frequency, of the two counter-propagating sine waves that created the standing wave.

Although it is created from two propagating waves, the standing wave pattern itself does not propagate.

Each element of the medium (say, the string) oscillates locally as a sinusoid in time : $\cos(\omega t)$ with an amplitude that is fixed for all time at that location, x. But from location-to-location, the amplitude of the oscillation varies as amplitude = $[2A \sin(kx)]$.

The location where the amplitude of the standing wave pattern is maximal is called an anti-node.

The location where the amplitude of the standing wave is zero is called a node.

There are three types of boundary conditions that support standing wave patterns : closed-closed, open-open, and open-closed.

A string held fixed at both ends, as in a guitar or violin, exhibits the closed-closed boundary condition.

A tube that is open at both ends, as in a flute or a toilet-paper roll, exhibits the open-open boundary condition.

A tube that is closed at one end, as in a clarinet or an uncapped plastic bottle, exhibits the open-closed condition.

A closed-closed condition requires a node at each end. The lowest (fundamental) mode of oscillation that can be sustained (a.k.a the first harmonic) has one antinode in the middle and has a wavelength that is twice the length of the string. The next higher mode has two antinodes and is called the second harmonic and is half a wavelength longer than the fundamental. Each higher harmonic adds an additional half-wavelength.

The harmonic frequencies are described by the formula : $f_n = nv/2L$; $n = 1, 2, 3, 4....$

An open-open condition requires an antinode at each end. The fundamental mode has one node in the middle and has a wavelength that is twice the tube length. The next higher mode has two internal nodes and is called the second harmonic and is half a wavelength longer than the fundamental. Each higher harmonic adds an additional half-wavelength. The harmonic frequencies are described by : $f_n = nv/2L$; $n = 1, 2, 3, 4....$

An open-closed condition requires a node at the closed end and an antinode at the open end. The fundamental mode has no internal nodes or antinodes and has a wavelength that is four times the length of the tube. The next higher mode (second harmonic) is half a wavelength longer (tube length = ¾ wavelength). Each higher harmonic adds an additional half-wavelength. The harmonic frequencies are described by the formula : $f_n = nv/4L$; $n = 1, 3, 5, 7....$

When two traveling waves of slightly different frequencies are superimposed on each other, the phenomenon of beat frequencies can be observed as a result of the two waves gradually drifting in-and-out of phase with each other at the location of the observer. The rhythmic modulation of the overall sound amplitude has a frequency equal to the difference of the two individual frequencies : $f_{beat} = | f_1 - f_2 |$

Electromagnetic (EM) Waves

Maxwell's Equations is a collection of 4 equations that form the basis of all electrical and magnetic phenomena and predict the propagation of electric or magnetic disturbances as an electromagnetic (EM) wave.

Electromagnetic waves travel through vacuum at the speed of light, given by : $c = 1 / \text{sqrt}(\mu_0\varepsilon_0) = 3 \times 10^8$ m/s

In an EM wave, the ratio of the amplitudes of the electric (E) and magnetic (B) fields is given by $c = E/B$.

An EM wave consists of an sinusoidal electric field oscillating perpendicular to an sinusoidal magnetic field. Both fields are also perpendicular to the direction of wave propagation.

The intensity of an EM wave is proportional to the square of the amplitude of either oscillating field.

An EM wave is an example of a non-mechanical transverse wave. It can travel through vacuum.

The intensity of an EM wave is equal to the Power carried by the wave divided by the area of its wavefront.

Visible light is an EM wave, and makes up a small portion of the full EM spectrum.

EM waves include (in order of decreasing increasing wavelength : Gamma rays, X-rays, Ultraviolet (UV) light, Visible light, Infrared (IR) light, microwaves, and radiowaves.

Visible light makes up a small fraction of the electromagnetic spectrum, and runs from 400 -700 nm wavelengths.

In a vacuum, all EM waves have the same speed, but have different wavelengths and frequencies

Violet/Blue light is at the short-wavelength end of the visible range. Red light is at the long-wavelength end.

The polarization of an EM wave describes the orientation of the electric field in the wave.

Light from thermal sources generally contain every orientation of polarization. This light is call “unpolarized”.

Light in which all of the electric fields point along a single line is called linearly polarized light.

Light of any polarization can be decomposed into a horizontal electric field component and a vertical component.

A linear polarizer passes (transmits) light that is polarized along the direction of its transmission axis.

A linear polarizer blocks (absorbs) light that is polarized perpendicular to its transmission axis.

“Unpolarized Light” can be turned into linearly polarized light by passing it through a linear polarizer.

When linearly polarized light is incident on a linear polarizer that has its transmission axis at an angle, θ, relative to the incident light's polarization, the transmitted fraction is given by Malus' Law : $I = I_o\cos^2\theta$.

Wave Optics

Young's double slit experiment demonstrated the wave nature of light by producing interference fringes on a screen when two closely spaced slits were illuminated by a coherent light source.

Two slit interference produces a bright fringe at points on the screen that have a difference in path lengths from the two slits that are equal to an integer multiple of a wavelength of the illuminating light.

Two slit interference produces a dark fringe at points on the screen that have a difference in path lengths from the two slits that are equal to a half (or 3/2 or 5/2) wavelength of the illuminating light.

In evaluating the positions of the fringes on the screen, we often use the small angle approximation. If we also assume that the two slits are infinitesimally thin, then the interference intensity pattern has the regularly-spaced form of a $\cos^2$ function

Light passing through a single rectangular slit produce a diffraction pattern, which is actually an interference pattern between light waves passing through different parts of a rectangular slit of finite width, *a*.

The locations of the maxima of the bright fringes of a single-slit diffraction pattern are difficult to calculate, but the exact center of the dark fringes can be easily calculated to be at the angles : $\sin\theta = m\lambda / a$

Light passing through a round opening produces a circularly symmetric diffraction pattern called an Airy pattern. The width of the central bright disk determined by the angle to the first dark fringe of the Airy pattern, which in turn determines the minimum angular resolution for imaging through a circular opening.

The limiting resolution for imaging through a rectangular slit of width, *a*, is $\theta_{min} = \lambda / a$.

The limiting resolution for imaging through a circular opening of diameter, *D*, is $\theta_{min} = 1.22 \cdot (\lambda / D)$.

A diffraction grating consists of a series of rectangular slits (or lines) that are equally spaced with a spacing that is on the order of the wavelength of the light incident on it.

Light incident on a diffraction grating will produce bright spots on a distant screen. The angle to these bright spots is the same as for the angle to the bright fringes due to illumination of two slits with the same spacing.

Compared to the two-slit interference pattern, the bright spots of the diffraction grating are more localized (more narrow), and the dark bands are wider.

Thin Films

Light waves (or other EM waves) travel through vacuum at the vacuum speed of light, $c = 3 \times 10^8$ m/s

When light waves (or other EM waves) travel through different media, the speed of the light is reduced by a factor, n, called the index of refraction of that material. $v_{light\_in\_media} = c / n$

When light waves (or other EM waves) travel through different media, the wavelength of the light is shortened by a factor of n (index of refraction) compared to its wavelength in vacuum. $\lambda_n = \lambda / n$

When light transitions from one medium to another medium of a different refractive index, a partial reflection occurs at the interface between the two media.

If light is moving from a medium with a lower index of refraction to a medium with a higher index of refraction, then the reflection will undergo a 180° (half-wavelength) phase shift upon reflection.

If light is moving from a medium with a higher index of refraction to a medium with a lower index of refraction, then the reflection will NOT undergo a phase shift.

Interference between the reflections off the front and back surfaces of a thin film of transparent material results in a maximum reflection of a particular wavelength if the reflection off the second interface arrives back at the first interface exactly in phase with the reflection off the first interface.

The relative phase of the second-surface reflection depends on : the path length through the film, the reduction of the wavelength within the media, and the (possible) phase inversion at the interfaces.

Ray Optics : Reflection and Refraction

In geometric optics, we ignore (most) of the wave properties of light and treat light as a ray that travels in straight lines through uniform media.

At an interface between two dissimilar media, light can undergo reflection or refraction.

Reflection at a smooth interface obeys the law of reflection : the incident angle is equal to the reflected angle.

When light transmits from one medium to another medium, the light ray can refract ("bend") to a new angle.

The amount that the light bends at an interface is determined by the values of the indices of refraction of the two materials that make up the interface, and is given by Snell's Law : $n_1 \cdot \sin\theta_1 = n_2 \cdot \sin\theta_2$, where the angle are measured relative to the normal to the interface.

Huygen's principle states that all points on a wavefront (the points of constant phase of the wave) can be treated as point sources of spherical "secondary wavelets" that travel outward at the speed of the wave in the medium.

The new wavefront at some later time can be found by drawing the tangent to these secondary wavelets

Huygen's principle, when applied to a wavefront incident (at an angle) on an interface between two media can explain the refraction ("bending") of light at that interface.

Although the index of refraction for a particular material is often given as a single number, it is actually a continuum of numbers that vary with the wavelength of the light. The variation of refractive index with wavelength is called "dispersion".

The angular spreading of different colors (wavelengths) of light through a prism is a combined result of dispersion and Snell's Law.

Most normal transparent materials have a dispersion curve in which the index of refraction is greater for shorter wavelengths of light. Thus, blue light "bends" more than red light as it transmits from air to glass.

When incident from a lower refractive index material (such as air) to a higher refractive index material (such as glass), all possible incident angles will result in some real transmitted angle in the higher index media.

When incident from a higher refractive index material (such as glass) to a lower refractive index material (such as air), there is a "critical angle" (measured relative to the normal to the interface) beyond which no light is transmitted into the lower index media. This phenomenon is called Total Internal Reflection.

The critical angle occurs when the transmitted angle in the lower index media reaches 90 degrees relative to the normal to the interface. Beyond the critical angle, Snell's Law does not result in a real angle solution for the transmitted beam, and thus no light actually transmits. All the light is reflected at the angle of reflection.

Image Formation by Lenses and Mirrors

An image is form when many (or all) of the light rays that leave a particular point either converge to another single point or appear to diverge from another single point that is at a different location from the object itself.

If the light rays actually converge to a point in space, then a real image of the original object is formed there.

If the light rays do not physically converge to a point, but appear to diverge from a common point that is separate from the original object, then a virtual image of the original object is formed at that point.

For a real image, a screen (piece of paper) placed at the image point of a glowing object would result in a sharp copy of that glowing object appearing on the screen.

For a virtual image, a screen (piece of paper) placed at the image point of a glowing object would NOT result in a sharp copy of that glowing object appearing on the screen.

Images can be produced by spherical lenses or spherical mirrors, both of which have surfaces that are a part of larger spherical surfaces with a particular radii of curvature.

The principal axis (a.k.a., optical axis) of a mirror or lens is the line that runs through the symmetrical center of the front face of the mirror/lens and is perpendicular to the surface at that point.

The focal length of a mirror or lens is the distance (from the lens or mirror) that a sharp image will form when the object is infinitely far away so that the incident rays are parallel. If the parallel rays are also parallel to the principal axis, then the rays will converge to the focal point, which also lies on the principal axis.

For a spherical mirror, the focal length is exactly half of the radius of curvature of the mirror.

For a spherical lens, the focal length depends on both the radii of curvature and the index of refraction. For a lens surrounded by air, the focal length is given by the the Lensmaker Formula : $1/f = (n-1)(1/R_1 - 1/R_2)$

The location of the image formed by a spherical lens or mirror is given by the Lens Equation : $1/f = 1/d_o + 1/d_i$

The magnification of an image formed by a thin lens or mirror is given by $m = h'/h = -d_i/d_o$.

Glass or plastic shaped with a bi-convex or plano-convex shape acts as positive lens (a.k.a. converging) lenses.

Glass or plastic shaped with a bi-concave or plano-concave shape acts as negative (a.k.a. diverging) lenses.

For light incident from the left on a lens, the positive object space is on the left of the lens, the positive image space is on the right of the lens, and the radius of curvature is positive if the center of curvature is to the right of the corresponding lens surface.

For a positive lens, an object out at infinity on the left of the lens produces an inverted, smaller, real image to the right of the lens.

As the object moves toward a positive lens from infinity, the real inverted object moves away from the lens to the right, and grows in height. When the object reaches twice the focal length, the real inverted image reaches the same height as the original and is located at a position of twice the focal length to the right of the lens.

As the object move inside the focal length of the positive lens, the image flips from being an larger, inverted, real image at the far right of the lens, to being a larger, upright, virtual image at the far left of the lens.

Image Formation by Lenses and Mirrors (cont)
[assume incident light is from the left]

Ray tracing of lenses and mirrors proceeds with the three following rules: (assume light incident from the left)

1) The ray leaving the object parallel to the principal axis passes through the secondary focal point
2) The ray leaving the object on a trajectory that passes through the primary focal point exits the lens parallel to the principal axis.
3) The ray leaving the object on a trajectory through the center point does not change its angle.

For a positive lens, the primary focal point is to the left of the lens and the secondary focal point is to the right.

For a negative lens, the focal length is negative, and the primary focal point is to the right of the lens while the secondary focal point is to the left of the lens.

For a positive (concave) mirror, there is only one focal point, and it is to the left of the mirror surface.

For a negative (convex) mirror, there is only one focal point, and it is to the right of the mirror surface.

For all thin spherical lenses, the center point for ray tracing is the center of the thin lens

For all spherical mirrors, the center point for ray tracing is at the center of curvature of the mirror.

For a system of two or more lenses, light from a light source on the left passing through a the first lens produces either a virtual image to the left of the lens or a real image to the right of the lens. This image then serves as a “second stage” object for the second lens, which also produces a real or virtual “image of the image”.

The Human Eye

Accommodation refers to the ability of the eye change the focal length of the crystalline lens of the eye by adjusting its shape.

When the ciliary muscles of the eye are relaxed, the crystalline lens has a more flat shape, and thus a longer focal length. This relaxed state is the least accommodated, and is used for viewing objects located far away.

When the ciliary muscles of the eye are strained, the crystalline lens has a more rounded shape and thus a shorter focal length. This is the most accommodated state and is used for viewing objects located nearby.

The far point of a person's eye is the furthest distance at which an object can be positioned and still form a sharp image on the retina with the unaided lens of the eye. The far point of a healthy human eye is at infinity.

The near point of a person's eye is the nearest distance at which an object can be positioned and still form a sharp image on the retina with the unaided lens of the eye. The near point of a healthy human eye is ~ 25.0 cm

A person with myopia is near-sighted. They can see nearby objects clearly, but cannot see clearly beyond their (myopic) far point. Their (myopic) far point is closer in than the far point of a healthy human eye.

A person with hyperopia is far-sighted. They can see faraway objects clearly, but cannot see clearly if an object is closer than their (hyperopic) near point. This (hyperopic) near point is farther out than that of a healthy eye.

The prescription lens for a person with myopia (near-sightedness) is a negative focal length lens. Its job is to take an object at the “normal far point” (infinity) and make an image of it at the person's myopic far point.

The prescription lens for a person with hyperopia (far-sightedness) is a positive focal length lens. Its job is to take an object at the “normal near point” (25 cm) and make an image of it at the person's hyperopic near point.

Compound Microscopes

A traditional microscope objective takes a sample placed just outside of its focal length and makes a real image of it at the “intermediate image plane” (typically ~160 mm behind the objective). This “intermediate image” acts as the object for an eyepiece that produces a virtual image at $-\infty$ so it can be viewed with a relaxed eye.

The resolution of a microscope is determined by the numerical aperture (NA) of the microscope objective and is given by the Abbe resolution limit : $r_{xy,Abbe} = (\lambda / 2 \cdot NA)$

The numerical aperture (NA) of a microscope objective lens depends on the maximum half-angle (center-to-edge) of light acceptance and on the index of refraction of the immersion medium ($NA = n \cdot \sin\theta_{max}$)

Quantum Mechanics (QM)

Blackbody radiation describes the spectrum of electromagnetic emission given off by warm objects.

The first quantum mechanical model arose as a way to correctly model the observed shape of the blackbody spectrum. (The classical wave model incorrectly predicted an explosion of energy in the UV portion of the spectrum - an inconsistency called the “UV catastrophe”)

A key feature of the quantum mechanical model is the introduction of discrete energy states.

Quantum mechanical effects become significant at very small scales. At macroscopic (large) scales, quantum mechanical effects are too small to be noticed. The requirement that quantum mechanics must correctly reproduce classical results at macroscopic scales is called the correspondence principle.

In the quantum mechanical model of EM waves, light can be described as a stream of discrete particles called photons. Each photon has a specific wavelength, frequency, and energy, related by $E = hf = hc/\lambda$

An important application of the QM model of light is the accurate description of the ejection of electrons from a metal surface exposed to certain incident light, a phenomenon called the photoelectric effect.

In the photoelectric effect, electrons are only emitted from the metal if the energy of the incident photons exceed the metal's work function ϕ (the binding energy that holds the outermost valence electrons to the metal), with the excess energy going into the kinetic energy of the emitted electron.

The kinetic energy of the emitted electron in the photoelectric effect depends on the energy (and thus the frequency or wavelength) of the incident photon, but does not depend on the intensity of the light beam. Increasing the intensity of the light beam increases the number of emitted electrons, but not their kinetic energy.

In the wave picture, intensity relates to the square of the wave amplitude. In the particle picture, intensity relates to the number (or density) of particles.

Light can legitimately be thought of both as a wave or as a particle. In some situations, light behaves more like a wave, and it other situations, light behaves more like a particle.

Compton scattering of X-rays off of electrons provided proof of the particle nature of EM waves. The scattered X-rays were observed to have a positive wavelength shift that depends only on the angle of scattered X-ray, and does not depend on the incident wavelength of the EM wave.

Compton scattering provided evidence that a photon is a particle with a distinct energy and momentum.

De Broglie correctly hypothesized that if something classically considered a wave (like light) has a particle nature, then classical particles should also have a wave nature and thus an associated wavelength.

The De Broglie wavelength of a particle (e.g., an electron) is inversely related to its momentum : $\lambda = h/p$.

The Heisenberg uncertainty principle states that the particle's position along an particular axis and the particle's momentum along that same axis cannot be simultaneously known with infinite accuracy. The product of the uncertainties cannot be smaller than the constant value $h/4\pi$: $\Delta x \cdot \Delta p_x \geq h/4\pi$

At the quantum mechanical level, a particle is described by a wave function, Ψ, which has an associated probability amplitude, ψ. The square of the probability amplitude is called the probability density, $|\psi|^2$, and gives the probability of the particle being found in a particle point in space.

If a microscopic particle is placed in a one-dimensional box, then the wavefunction must obey the boundary conditions of the box : the probability of finding the particle outside the box must be zero, which implies that ψ must go to zeros at the edges of the box. These boundary conditions give rise to solutions that are sinusoids of discrete wavelengths. This is turn gives rise to a discrete set of possible energies states for the trapped particle. $E_n = (h^2 n^2) / (8mL^2)$ $n = 1,2,3,...$

Atomic Physics

Prior to 1911, scientists (incorrectly) believed in a “Plum pudding” model of an atom.

In 1911, Ernest Rutherford discovered the atomic nucleus as a concentrated positive charge at the center of the atom; but his planetary model of the atom incorrectly imagined the electrons circling the nucleus in planet-like orbits.

The (incorrect) planetary model of the atom would exhibit continuous, gradual orbital decay of the electron orbit into the nucleus; this does not happen, and so the planetary model is incorrect.

The planetary model was replaced by Bohr's semi-classical (and semi-quantum-mechanical) model of the atom in which only specific, discrete electron orbits that were stable and did not decay or radiate except when jumping to another allowed discrete orbit.

The Bohr model of the atom was replaced with the (currently accepted) fully quantum mechanical model of the atom, which correctly predicts the splitting of spectral lines into doublets and triplets and also accurately populates the periodic table of elements.

In the full QM model of the atom, each electron occupies a unique electronic state that is described by four quantum numbers :

n - principal quantum number : describes the energy level (or “shell”) of the electron

ℓ - orbital quantum number : describes the orbital angular momentum (or “subshell”) of the electron

m_ℓ - orbital magnetic quantum number : describes the azimuthal component of the angular momentum (or “orbital”) of the electron.

m_s - spin magnetic quantum number : describes the intrinsic spin of the electron.

The quantum numbers only take on specific values, some depending on the value of other quantum numbers:

n = 1,2,3,4,.... traditional names : 1 = K , 2 = L , 3 = M , 4 = N , 5 = O , ….

ℓ = 0, 1, ...(n-1) traditional names : 0 = s , 1 = p , 2 = d , 3 = f

$m_\ell = -\ell, -\ell+1, \ldots 0, \ldots \ell-1, \ell$

$m_s = -½ , + ½$

The Pauli exclusion principle states that each unique quantum state can only be occupied by one electron.

For a hydrogen atom, the energy associated with a quantum state depends on its principal quantum number and is given by $E_n = -13.606 \text{ eV} / n^2$ (n = 1, 2, 3, ….)

The magnitude of the orbital angular momentum of the hydrogen atom is quantized and can only take on the discrete values : $L = \sqrt{l(l+1)}\hbar$ (ℓ = 0,1,2,... n-1)

The angle that the orbital angular momentum vector makes with respect to an external magnetic field is also quantized and can only take on discrete values (this effect is called space quantization). As a result, the azimuthal component (z-component) of the orbital angular momentum is also quantized : $L_z = m_l\hbar$

The lowest energy state (n=1) is called the ground state.

The ground state energy is negative, and its magnitude corresponds to the ionization energy for that atom.

When an electron in an atom moves from a higher energy state to a lower energy state, it emits a photon with an energy equal to the difference between the two energy states.

An electron can move from a lower energy state to a higher energy state in an atom by absorption of a photon with an energy equal to the difference between the two energy states.

The Lymann, Balmer, and Paschen spectral lines of hydrogen correspond to transitions from higher energy states to the n=1, n=2, and n=3 shells respectively. The transition wavelength is given by the (generalized) Rydberg formula : $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$, where R is the Rydberg constant is $R = 1.097 \times 10^7 m^{-1}$

Nuclear Physics

The nucleus of an atom is composed of nucleons (a general term for either protons or neutrons).

A particular nuclide is characterized by a specific number of protons and neutrons and denoted by ${}_Z^A X$.

In a nuclide, "A" is called the "mass number" and represent the total number of nucleons

In a nuclide, "Z" is called the "atomic number" and represents the total number of protons

In a nuclide, the total number of neutrons is calculated by N = A-Z, and is called the "neutron number"

Different atomic elements have different Z values.

Different isotopes of the same element have the same Z value, but different N values (and thus different A=N+Z)

The strong nuclear force and the weak nuclear force are responsible for holding the positive charged nucleus together against the mutual repulsion of the Coulomb force, and responsible for the interconversion of protons and neutrons during beta decay, respectively.

Alpha decay is the emission of a Helium nucleus (a.k.a., an alpha particle) consisting of two neutrons and two protons, from a large unstable nucleus.

There are three distinct types of beta decay : β^-, β^+, and electron capture

Beta-minus (β^-) decay is the emission of an electron (e^-) and accompanied by the conversion of a neutron into a proton ($n \rightarrow p$), along with an accompanying emission of an electron antineutrino ($\overline{\nu}$).

Beta-plus (β^+) decay is the emission of a positron (a.k.a, an anti-electron, e^+) and accompanied by the conversion of a proton into a neutron ($p \rightarrow n$), along with an accompanying emission of an electron neutrino (ν).

Electron capture is a process that involves the nucleus capturing an inner-shell electron and is accompanied by the conversion of a proton into an electron, along with the accompanying emission of an electron neutrino (ν).

Gamma decay is the emission of a high energy photon (a gamma ray) from a nucleus that is an excited state, typically immediately after undergoing alpha or beta decay.

The mass-energy equivalence formula, $E = mc^2$, states that mass has an equivalent energy associate with it.

The mass of a nucleus is generally less than the summed mass of the individual nucleons that it is composed of. The energy associated with this mass difference is the binding energy of the nucleus and represent the energy required to break the nucleus apart into its individual nucleons (or alternatively, the energy release when the nucleus is assembled from its components by nuclear fusion).

Unstable atomic nuclei undergo radioactive decay in a probabilistic fashion. The exact time for any particular unstable nuclei to decay cannot be predicted, but for large populations of nuclei, the overall rate of decay and the number of remaining unstable nuclei can be modeled by an exponential function.

If we start with an initial number of unstable nuclei, N_0, at the initial time t=0, then the number of remaining unstable nuclei at time t>0 is given by : $N(t) = N_0\, e^{-\lambda t}$, where λ is the decay constant for this decay process.

The half-life, $T_{1/2}$, of a radioactive process is the time required for half of the initial nuclei to decay and is related to the decay constant by : $T_{1/2} = (\ln 2) / \lambda$

The radioactive activity, (a.k.a, the decay rate, R) describe the instantaneous rate of change in the number of unstable nuclei : $R(t) = |dN(t) / dt| = N_0 \lambda e^{-\lambda t} = R_0\, e^{-\lambda t}$.

If we wait one half-life, both the rate of decay and the number of remaining unstable nuclei are reduced by a factor of two. After two half-lives, both are reduced by a factor of $2^2 = 4\times$ compared to their original values, after three half-lives, both are reduced by a factor of $2^3 = 8\times$ compared to their original values.