**Geometry**

Here we review some basic geometric relations that are related to problems that will be seen in Physics 1A.

Key concepts:
- Angles along a straight line add up to 180°
- The angle between two perpendicular lines is 90° (right angle)
- Intersecting lines produce opposite angles of equal size (congruent angles)
- The sum of interior angles in any n-sided polygon add up to (n-2)x180° (e.g. 3-sided triangle = 180°, 4-sided square = 360°, etc.)

1. Here we want to review some basic geometric relations that are related to problems that will be seen in Physics 1A. Consider the following angles in the figure shown. The horizontal lines are parallel.
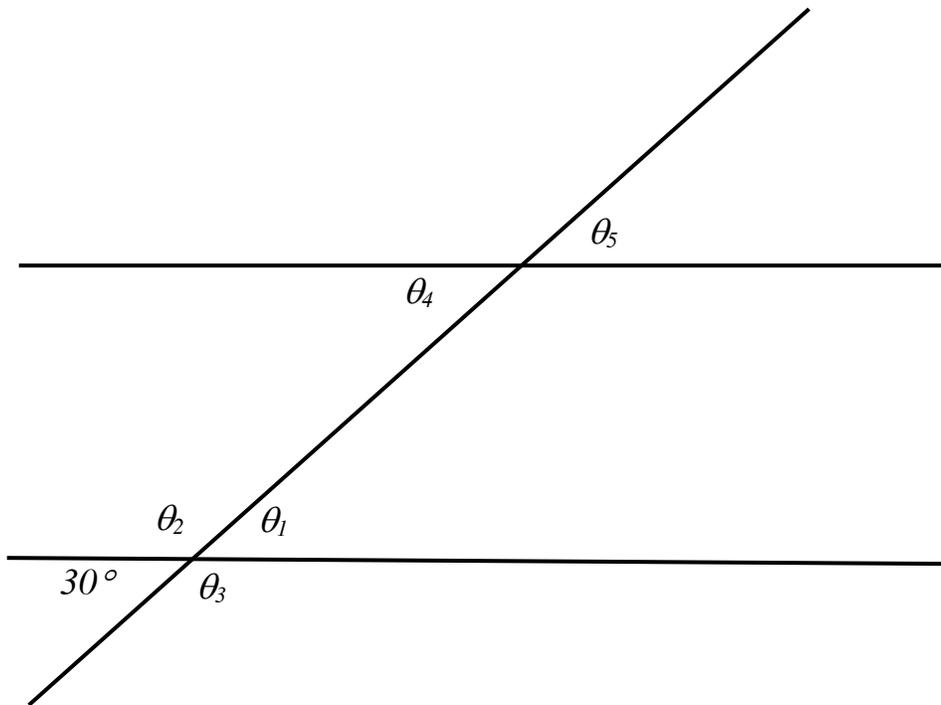
Given the angle provided, can you determine the other angles indicated? What rules apply?
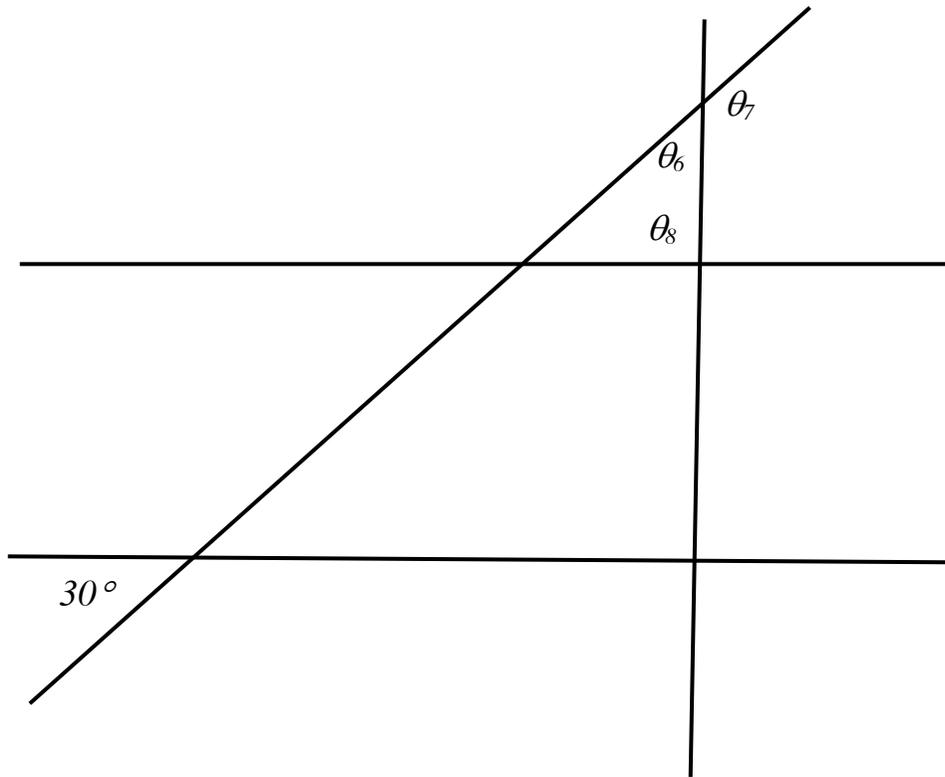
$\theta_1 =$ $\theta_2 =$

$\theta_3 =$ $\theta_4 =$

$\theta_5 =$

2. Now let's add another line that is perpendicular to the parallel lines as shown. Everything else is the same as that in question 1 of this section. What are the values for the angles indicated and what rules apply?


$\theta_6 =$

$\theta_7 =$

$\theta_8 =$

3. Now let's add yet another line that crosses from upper left to lower right through the previous set of lines as shown. The line creates an angle of 70° with respect to the first bisecting line from part 1 (as shown, not quite to scale). Everything else is the same as that in questions 1 and 2 of this section. What are the values for the angles indicated and what rules apply?
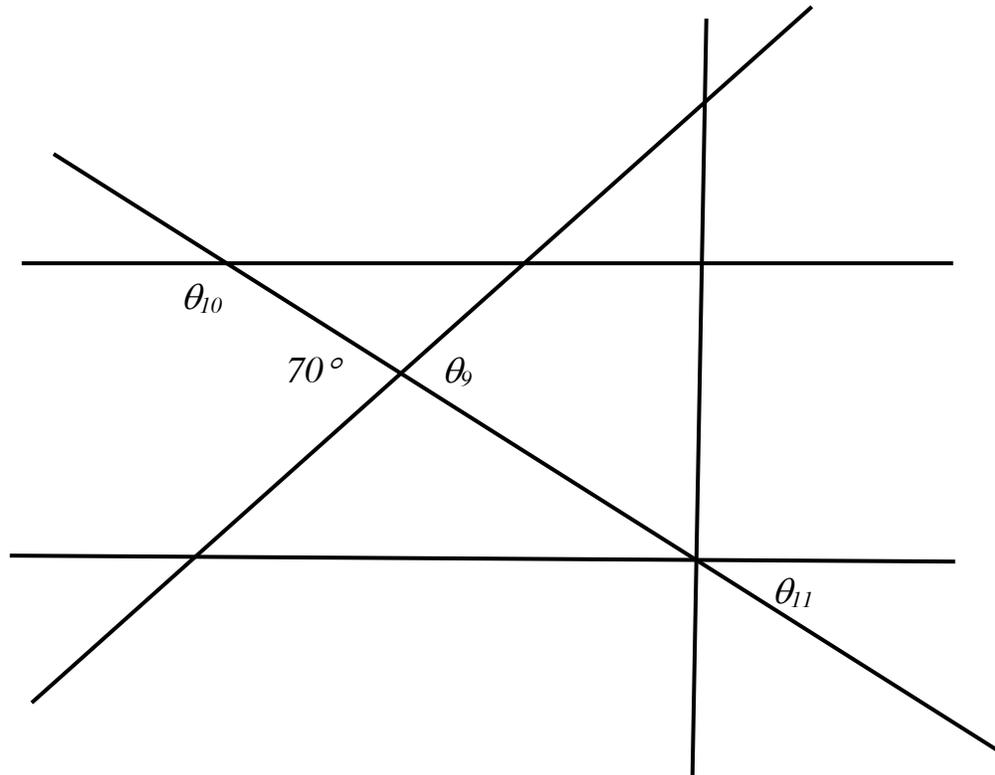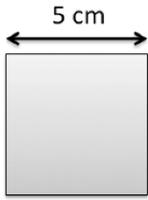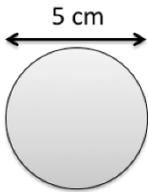

$\theta_9 =$
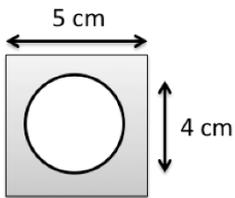
$\theta_{10} =$

$\theta_{11} =$

4. What are the areas of the following objects?
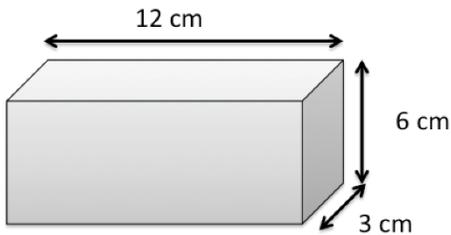
5 cm

Square with side length 5 cm:

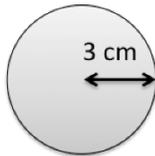5 cm

Circle with diameter 5 cm:

5 cm

4 cm

Square with side length 5cm and a
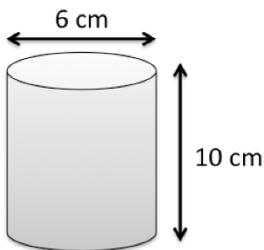4 cm diameter circular hole:

5. What are the volumes of the following objects?

12 cm

6 cm

3 cm

A rectangular box with sides 12
cm, 6 cm and 3 cm:

3 cm

A sphere with radius 3 cm:

6 cm

10 cm

A cylinder with circular diameter
6 cm and height 10 cm:

Trigonometric Functions

6. Here we want to review some basic properties of trigonometric functions. These will be important for determining the *components* of vectors such as velocity, acceleration and forces.

    Key concepts:
    - In a right triangle:
      - $\sin(\theta)$ = opposite/hypotenuse
      - $\cos(\theta)$ = adjacent/hypotenuse
      - $\tan(\theta) = \sin(\theta)/\cos(\theta)$ = opposite/adjacent
      - Can be remembered with mnemonic SOHCAHTOA
    - In a right triangle, the perpendicular side lengths A and B and hypotenuse length C are related as $A^2 + B^2 = C^2$ (Pythagorean theorem)
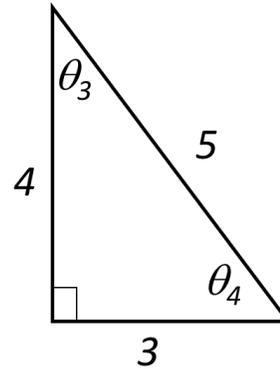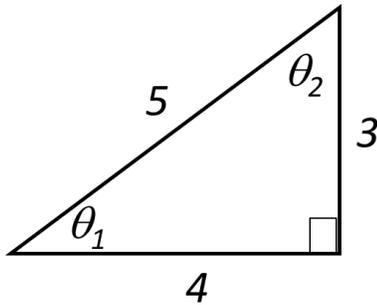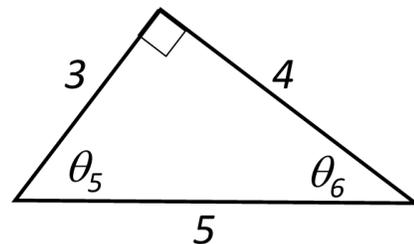

Consider the two right triangles shown above. Provide the values for the following trigonometric functions and parameters:

$\cos(\theta_1) =$ _____ $\cos(\theta_4) =$ _____

$\sin(\theta_1) =$ _____ $\sin(\theta_4) =$ _____

$\tan(\theta_1) =$ _____ $\tan(\theta_4) =$ _____

$\theta_1 =$ _____ $\theta_4 =$ _____

7. For the right triangle shown below, provide the values for the following trigonometric functions and angles:

    $\sin(\theta_5) =$ _____
    $\cos(\theta_5) =$ _____
    $\sin(\theta_6) =$ _____
    $\tan(\theta_6) =$ _____
    $\theta_5 =$ _____
    $\theta_6 =$ _____

8. For the triangle shown to the right, with
$\theta_6 = 33^0$ and $C = 7$ m,
what are
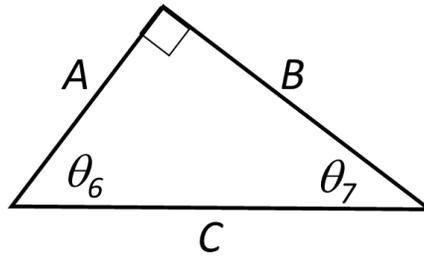
$A =$ _____

$B =$ _____

$\cos(\theta_7) =$ _____

$\theta_7 =$ _____

Vectors

Here we review some basic vector properties and operations, particularly important for vector quantities such as velocity, acceleration and forces

<u>Key concepts:</u>
- Vectors can be rewritten in component notation $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j}$, where A_x and A_y are the projects of A onto the x and y axes
- The length (magnitude) of a vectors $|\mathbf{A}| = (A_x^2+A_y^2)^{1/2}$
- Vectors can be added together by adding their components: $\mathbf{A} + \mathbf{B} = (A_x+B_x)\mathbf{i} + (A_y+B_x)\mathbf{j}$
- Vectors can be scaled by distributing the factor to the components: $c\mathbf{A} = cA_x\mathbf{i} + cA_y\mathbf{j}$
- The dot product computes the projection of one vector onto another: $\mathbf{A} \bullet \mathbf{B} = A_xB_x+A_yB_y$ $= |\mathbf{A}||\mathbf{B}|\cos\theta$, where θ is the angle between the vectors
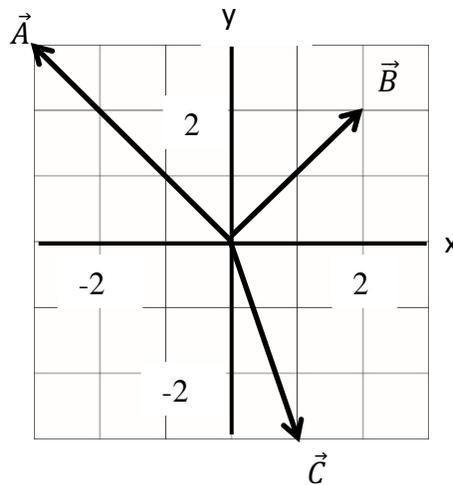
9. Consider the three vectors shown above. Write the vectors in terms of the unit vectors for the x- an y- directions ($\hat{\imath}$ and $\hat{\jmath}$):

$$\vec{A} = (\qquad)\,\hat{\imath} + (\qquad)\,\hat{\jmath}$$

$$\vec{B} = (\qquad)\,\hat{\imath} + (\qquad)\,\hat{\jmath}$$

$$\vec{C} = (\qquad)\,\hat{\imath} + (\qquad)\,\hat{\jmath}$$

$$\vec{A} + \vec{B} = (\qquad)\,\hat{\imath} + (\qquad)\,\hat{\jmath}$$

$$\vec{A} + \vec{C} = (\qquad)\,\hat{\imath} + (\qquad)\,\hat{\jmath}$$

$$\vec{A} - \vec{B} = (\qquad)\,\hat{\imath} + (\qquad)\,\hat{\jmath}$$

10. Plot the calculated vectors $\vec{A} + \vec{B}$, $\vec{A} + \vec{C}$ and $\vec{A} - \vec{B}$ on the graph below:

y

x
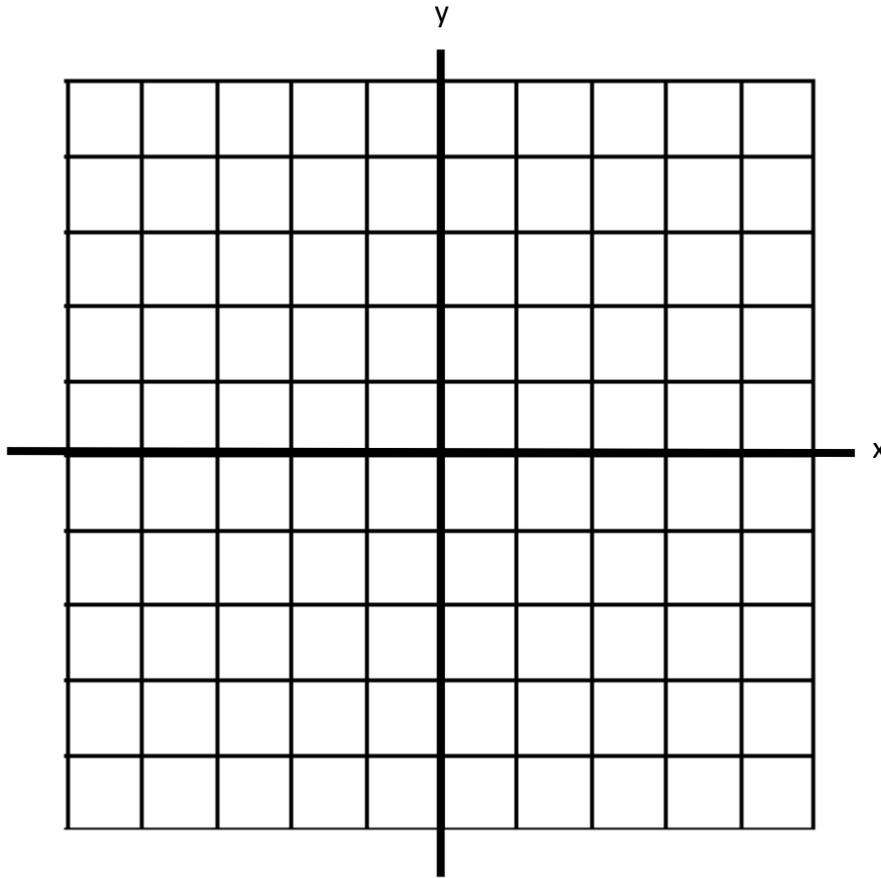
11. Rewrite the following vectors in terms of the magnitude and angle with respect to the x-axis:

$$\left| \vec{A} + \vec{B} \right| = \qquad ; \ \theta =$$

$$\left| \vec{A} + \vec{C} \right| = \qquad ; \ \theta =$$

$$\left| \vec{A} - \vec{B} \right| = \qquad ; \ \theta =$$

12. Compute to following dot products:

A • B =

A • C =

(A-B) • C =

Calculus

Here we review basic calculus that will be important for determining, for example, velocity as displacement over time, work as a function of which is force over distance, etc.

Key concepts:
- Derivative measures the slope of a function
- Integration measures the accumulation of a function (area under a curve)
- Important functions (A, k, and c are constants):
  - polynomials: $d/dx \, Ax^n = nAx^{n-1}$, $\int Ax^n \, dx = (A/n+1)x^{n+1} + c$ (for $n \neq 1$)
  - sine: $d/dx \, Asin(kx) = Akcos(kx)$, $\int Asin(kx) \, dx = -(A/k)cos(kx) + c$
  - cosine: $d/dx \, Acos(kx) = -Aksin(kx)$, $\int Acos(kx) \, dx = (A/k)sin(kx) + c$
  - exponential: $d/dx \, Ae^{kx} = Ake^{kx}$, $\int Ae^{kx} \, dx = (A/k)e^{kx} + c$
  - logarithm: $d/dx \, Aln(x) = Ak/x$, $\int A/x \, dx = Aln(x) + c$

13. For the function f shown in the upper graph, generate the derivative (df/dx) in the second graph below:
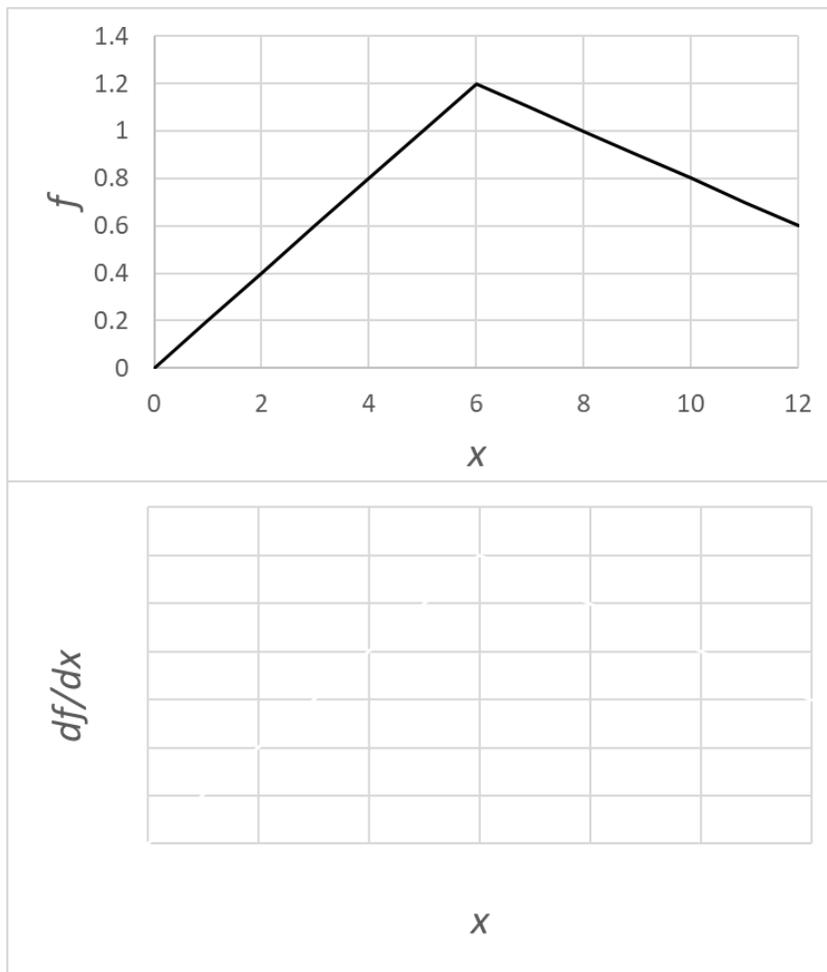
14. For the function f shown in the upper graph, generate the integral ($\int f dx$) in the second graph below:
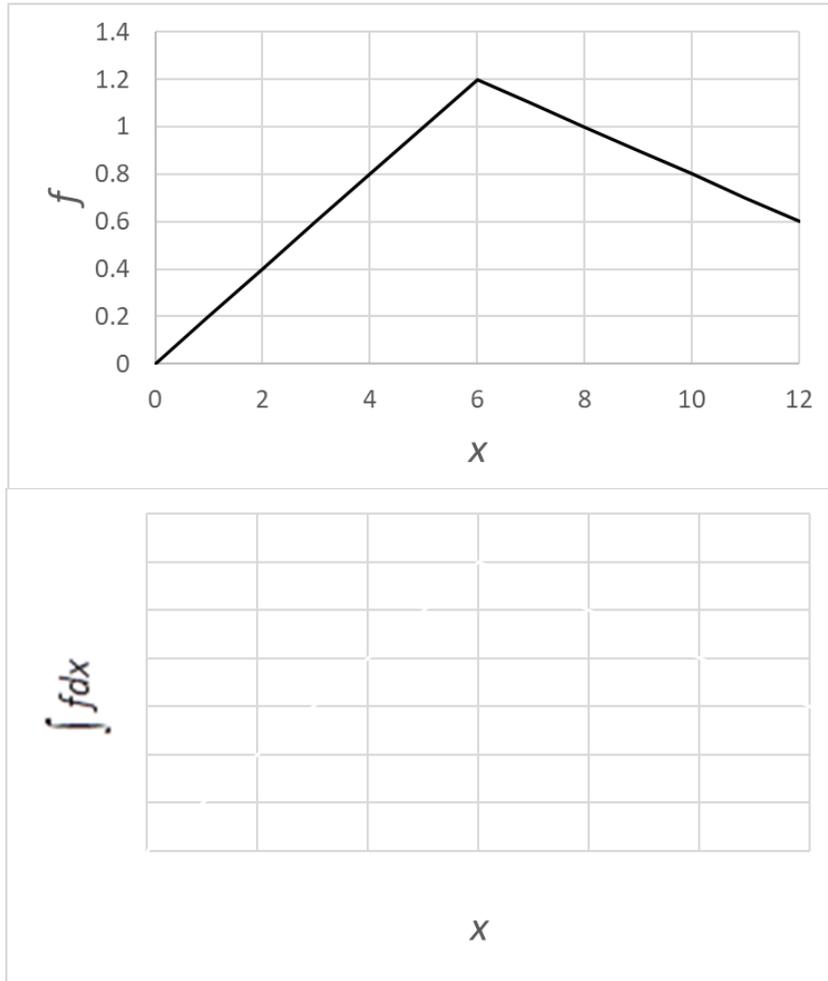
Calculate the derivative of the following functions:

$f = 5x$, df/dx = _____

$f = 5x^2$, df/dx = _____

$f = 5x^2 + 4$, df/dx = _____

Calculate the integral (indefinite) of the following functions:

$f = 5x$, $\int f dx$= _____

$f = 5x^2$, $\int f dx$= _____

$f = 5x^2 + 4$, $\int f dx$ = _____

Scaling (Quadratic Forms)

There are instances in Physics 1 where the functional form of an expression goes as a parameter squared. One example you will encounter in Physics 1A is that of the potential energy of a spring system which goes as $PE = \frac{1}{2} k x^2$, where x is the displacement from equilibrium. In this exercise, we will explore the consequences of having a computed value that is proportional to the square of a given or measured parameter.

15. Let's consider the example of the potential energy of the spring.
   (a) In the table provided, in column 2 calculate the PE for the values given in the first column for k = 1.

x	$PE = 1/2\, kx^2$	ΔPE
-6		
-5		
-4		
-3		
-2		
-1		
0		
1		
2		
3		
4		
5		
6		

   (b) In the graph below, plot the data over the range provided. Determine the best values for the domain (x values) and range (y values) before plotting. Sketch out the functions given the values in the table.
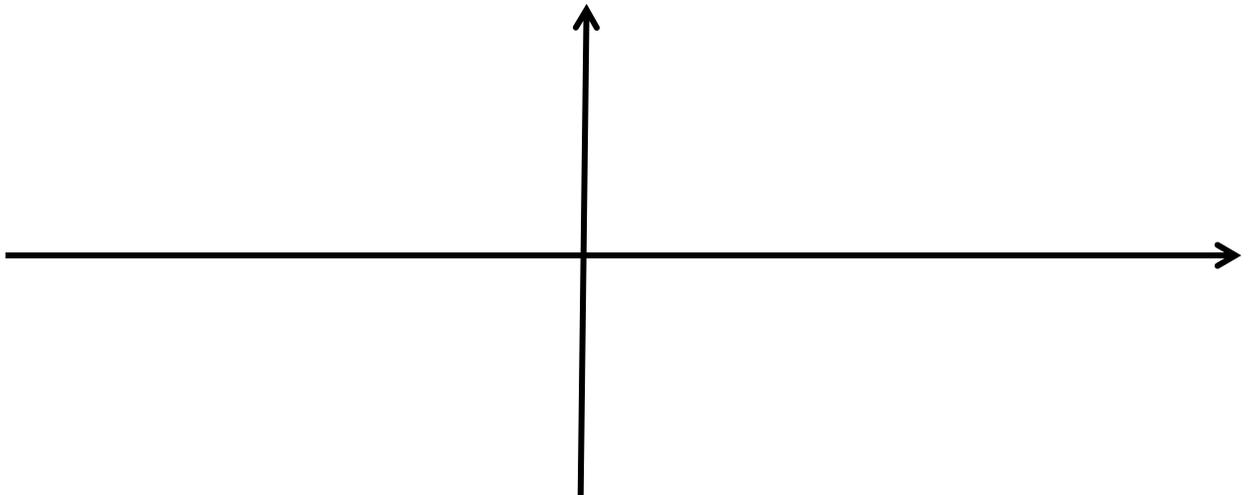
At what pair of x values are the PE the same?_____
(can you generalize?)

(c) For the calculation of the PE, if you double the value for x, how much larger is the value for the new PE' (*ie*. what is PE'/PE)?

(d) If you double the value for k, how much larger is the value for the new PE' (*ie*. what is PE'/PE)?

(e) Compare the *difference* in the PE between consecutive values of x and plot in column 3 of the table. How does the PE *difference* between $x = 2$ and $x = 1$ compare to the *difference* between $x = 1$ and $x = 0$?

(f) How does the PE *difference* between $x = 3$ and $x = 2$ compare to the *difference* between $x = 2$ and $x = 1$?